CHAPTER 26

CAPACITORS IN PARALLEL AND IN SERIES

It would be impossible to manufacture or to store capacitors of every conceivable value. As with resistors, capacitors in general use are manufactured to preferred values along with their tolerances. There is no universal colour coding of small capacitors at present, many companies using their own systems. Capacitors which are large enough have their capacitance, tolerance and maximum operating voltage stamped on the outside surface. By reading the values of capacitance, the capacitors may be connected in parallel or series to produce a capacitor of required value.

26.1 CAPACITORS IN PARALLEL

Two capacitors connected in parallel to a d.c. source are shown in figure 26.1.

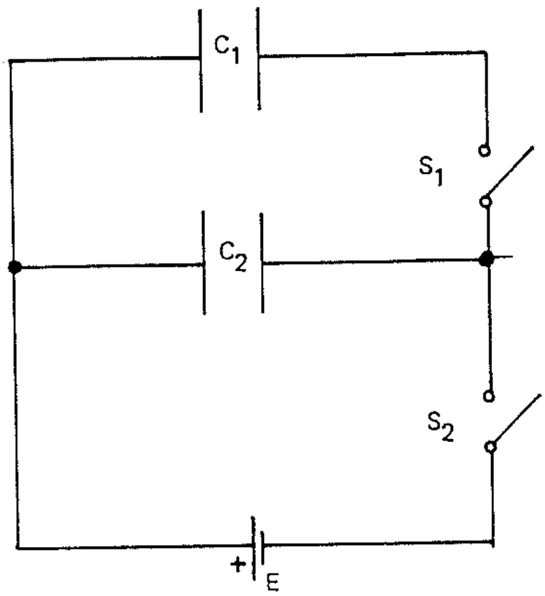

Figure 26.1

When the capacitors are completely discharged there will be a store of electrons on the negative side of the battery far in excess of the electrons on the capacitor plates.

If S_2 is closed there will be a rush of electrons to the capacitor, as explained in Chapter 25, and C_2 will become charged. (Figure 26.2).

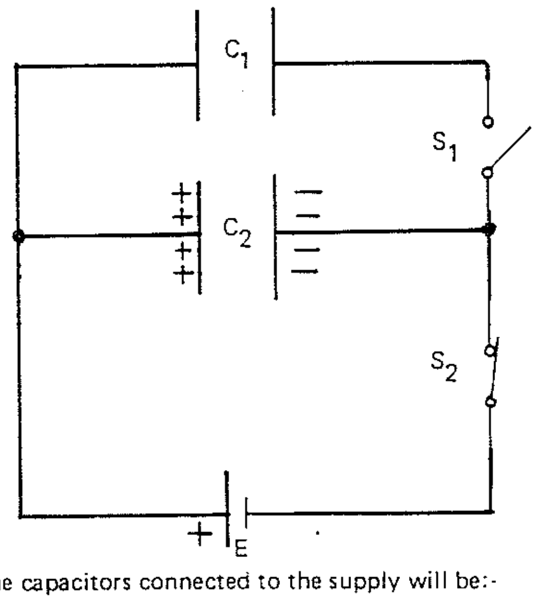

Figure 26.2

The charge on the capacitors connected to the supply will be:-

$Q_2 \quad = \quad C_2 E$ coulombs

Closing S_1 (figure 26.3) will cause C_1 to become charged.

$Q_1 \quad = \quad C_1 E$

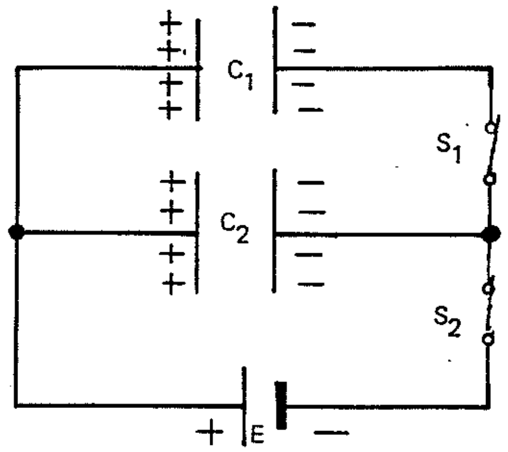

Figure 26.3

The total charge of the parallel circuit will be the sum of the individual charges.

$$Q = Q_1 + Q_2$$

As the only change in the circuit when S_1 is closed is the addition of another capacitor, the increase in charge must be due to an increase in capacitance.

$$C = C_1 + C_2.$$

Paralleling capacitors has the effect of increasing the plate size.

Ignoring the resistance of the connecting wires it can be seen that the potential at the plates of each capacitor will be the same as that of the source (after the current has decreased to zero). (Figure 26.4).

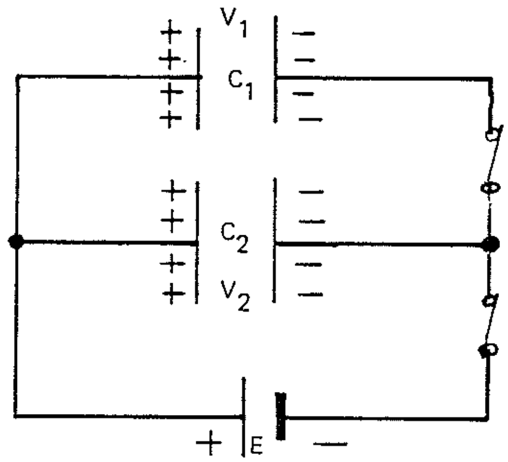

Figure 26.4

$$E = V_1 = V_2$$

The charge on a capacitor is given by $Q = CE$, so the total charge on the parallel bank of capacitors is -

$$CE = C_1 V_1 + C_2 V_2$$

But $E = V_1 = V_2$

So $Q = C_1 E + C_2 E$

Example 26.1

A 30 microfarad and a 20 microfarad capacitor are connected in parallel to a 500 volt d.c. supply. Determine —

(a) total capacitance

(b) charge on each capacitor

(c) total charge of the bank of capacitors

$C_1 = 30\ \mu F$

$C_2 = 20\ \mu F$

$E = 500\ V$

$C = ?\ \mu F$

$Q_1 = ?$ coulomb

$Q_2 = ?$ coulomb

$Q = ?$ coulomb

$C = C_1 + C_2$

$= (30 \times 10^{-6}) + (20 \times 10^{-6})$

$= 50\ \mu F$

$Q_1 = C_1 V_1$

$= 30 \times 10^{-6} \times 500$

$= 0.015$ coulombs

$Q_2 = C_2 V_2$

$= 20 \times 10^{-6} \times 500$

$= 0.01$ coulombs

$Q = CE$

$= 50 \times 10^{-6} \times 500$

$= 0.025$ coulombs

Check

$Q = Q_1 + Q_2$

$= 0.015 + 0.01$

$= 0.025$ coulombs

Example 26.2

Three capacitors C_1, C_2 and C_3 have capacitances of 25 μF, 50 μF and 100 μF respectively. The capacitors are connected in parallel to a 600 volt supply. Calculate:-

(a) total capacitance

(b) total charge stored

(c) charge stored by each capacitor.

$C_1 = 25\ \mu F$

$C_2 = 50\ \mu F$

$C_3 = 100\ \mu F$

$E = 600\ V$

$C = ?\ \mu F$

$Q = ?\ C$

$Q_1 = ?\ C$

$Q_2 = ?\ C$

$Q_3 = ?\ C$

(a) $C = C_1 + C_2 + C_3$

$= 25 + 50 + 100$

$= 175\ \mu F$

(b) $Q = CE$

$= 175 \times 10^{-6} \times 600$

$= 0.105\ C$

(c) $Q_1 = C_1 V_1$

$= 25 \times 10^{-6} \times 600$

$= 0.015\ C$

$Q_2 = C_2 V_2$

$= 50 \times 10^{-6} \times 600$

$= 0.03\ C$

$Q_3 = C_3 V_3$

$= 100 \times 10^{-6} \times 600$

$= 0.06\ C$

Check

$Q = Q_1 + Q_2 + Q_3$

$= 0.015 + 0.03 + 0.06$

$= 0.105\ C$

26.2 CAPACITORS IN SERIES

A circuit illustrating capacitors connected in series is shown in figure 26.5.

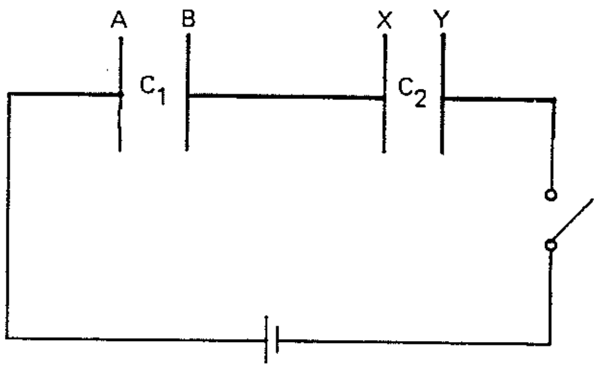

Figure 26.5

When the switch is closed there will be a rush of electrons from the source to plate 'Y' in capacitor C_2. (Figure 26.6).

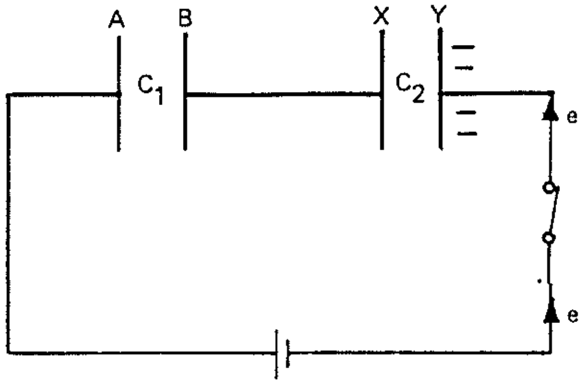

Figure 26.6

The extra electrostatic charge on plate 'Y' will repel an equal charge away from plate 'X' to plate 'B', leaving plate 'X' positively charged. The electrostatic charge on plate 'B' will repel an equal number of electrons away from plate 'A' back to the source, leaving plate 'A' positively charged. (Figure 26.7).

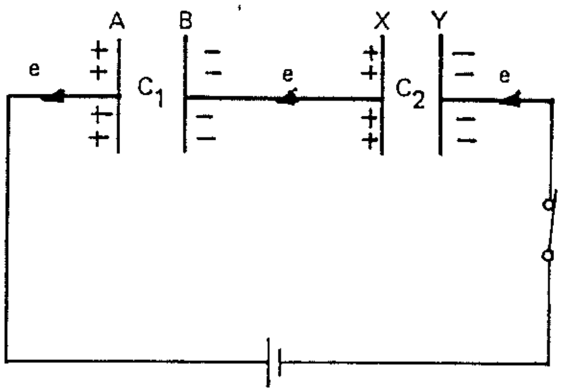

Figure 26.7

The movement of electrons to and from the plates and in the connecting wires has been the same throughout the circuit. Comparing the positive and negative charges between plates 'A' and 'B', 'X' and 'Y' and 'A' and 'Y' it can be seen that the states of charge are identical. From this it can be deduced that the charge stored by the complete series bank of capacitors (between plates 'A' and 'Y') is equal to the charge on each capacitor.

$$Q = Q_1 = Q_2$$

As the capacitors may have different size plates, dielectrics or distances between the plates. equal charges does not imply equal capacitances.

Applying Kirchhoff's voltage law around the circuit in gives $E = V_1 + V_2$ (Figure 26.8).

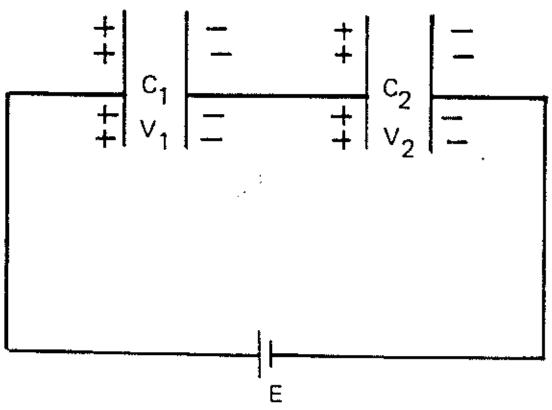

Figure 26.8

If the total capacitance of the group is taken as C, then the total charge will be:-

$$Q = CE$$

$$\text{or} \quad E = \frac{Q}{C}$$

$$\text{now} \quad E = V_1 + V_2$$

replacing the voltages in the Kirchhoff's law equation with the respective charges and capacitances —

$$\frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

But $Q = Q_1 = Q_2$ and dividing each term by Q leaves —

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or total capacitance

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

Example 26.2

Capacitors of 30 microfarad and 15 microfarad are connected in series to a 500 volt d.c. supply. Calculate -

(a) total capacitance.

(b) charge on each capacitor.

(c) total charge on the bank of capacitors.

(d) voltage drop across each capacitor.

$C_1 = 30\ \mu F$

$C_2 = 15\ \mu F$

$E = 500\ V$

$C = ?\ \mu F$

$Q_1 = ?$ coulomb

$Q_2 = ?$ coulomb

$Q = ?$ coulomb

$V_1 = ?\ V$

$V_2 = ?\ V$

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$= \frac{1}{\frac{1}{30} + \frac{1}{15}}$$

$$= \frac{1}{\frac{2 + 4}{60}}$$

$$= \frac{1}{\frac{1}{10}}$$

$$= 10\ \mu F$$

$$Q = CE$$

$$= 10 \times 10^{-6} \times 500$$

$$= 0.005 \text{ coulomb}$$

$$Q_1 = Q$$

$$= 0.005 \text{ coulomb}$$

$$Q_2 = Q$$

$$= 0.005 \text{ coulomb}$$

$$V_1 = \frac{Q_1}{C_1}$$

$$= \frac{0.005}{30 \times 10^{-6}}$$

$$= 166.6 \text{ V}$$

$$V_2 = \frac{Q_2}{C_2}$$

$$= \frac{0.005}{15 \times 10^{-6}}$$

$$= 333.3 \text{ V}$$

check 500 V = 166.6 V + 333.3 V

Example 26.4

Three capacitors C_1, C_2 and C_3 having capacitance of 8 μF, 12 μF and 24 μF respectively, are connected in series to a 400 volt supply. Determine:-

(a) total charge stored

(b) p.d. across each capacitor

(c) charge stored on each capacitor.

$C_1 = 8\ \mu F$

$C_2 = 12\ \mu F$

$C_3 = 24\ \mu F$

$E = 400$ V

$V_1 = ?$ V

$V_2 = ?$ V

$V_3 = ?$ V

$Q = ?$ C

$Q_1 = ?$ C

$Q_2 = ?$ C

$Q_3 = ?$ C

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{8} + \frac{1}{12} + \frac{1}{24}$$

$$= \frac{3 + 2 + 1}{24}$$

$$C = 4\ \mu F$$

(a) $$Q = C E$$

$$= 4 \times 10^{-6} \times 400$$

$$= 0.0016 \text{ C}$$

(b) $$V_1 = \frac{Q}{C_1}$$

$$= \frac{0.0016}{8 \times 10^{-6}}$$

$$= 200 \text{ V}$$

$$V_2 = \frac{Q}{C_2}$$

$$= \frac{0.0016}{12}$$

$$= 133\ 1/3 \text{ V}$$

$$V_3 = \frac{Q}{E_3}$$

$$= \frac{0.0016}{24}$$

$$= 66\ 2/3 \text{ V}$$

Check $E = V_1 + V_2 + V_3$

$= 200 + 166\ 1/3 + 66\ 2/3$

$= 400$ V

(c) $Q = Q_1 = Q_2 = Q_3$

$= 0.0016$ C

SUMMARY

In a bank of parallel connected capacitors —

$$C = C_1 + C_2$$

$$E = E_1 = E_2$$

$$Q = Q_1 + Q_2$$

In a bank of series connected capacitors —

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$E = E_1 + E_2$$

$$Q = Q_1 = Q_2$$

TUTORIALS 1.26

(1) Three capacitors of 9 μF, 12 μF and 18 μF are connected in series to a 400 V d.c. supply. Calculate the charge on the group of capacitors.

(2) Three capacitors whose capacitances are 10 μF, 15 μF and 25 μF are connected in parallel to a 500 V d.c. supply. Calculate the charge on the bank of capacitors.

(3) Three capacitors C_1, C_2 and C_3 having capacitance of 4 μF, 6 μF and 12 μF respectively are connected in series to a 500 V d.c. supply. Calculate:-

(a) total charge on the bank of capacitors

(b) voltage across each capacitor

(4) Three capacitors C_1. C_2 and C_3 of 5 μF, 15 μF and 25 μF respectively are connected in parallel to a 200 volt d.c. supply. Calculate:-

(a) total charge on the capacitor bank

(b) charge on each capacitor.

(5) Three capacitors connected in series to a 240 volt d.c. supply have a total charge of 0.0012 coulombs. If two of the capacitors have capacitance of 10 μF and 15 μF calculate the capacitance of the third capacitor.